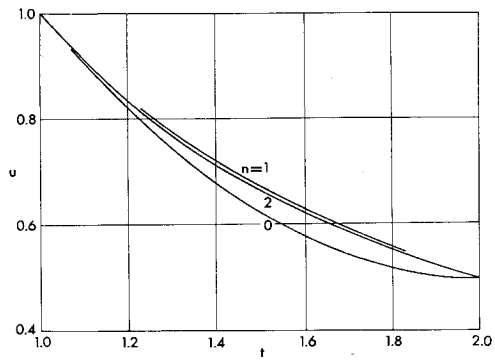

Fig. 4 The function $u(t)$.

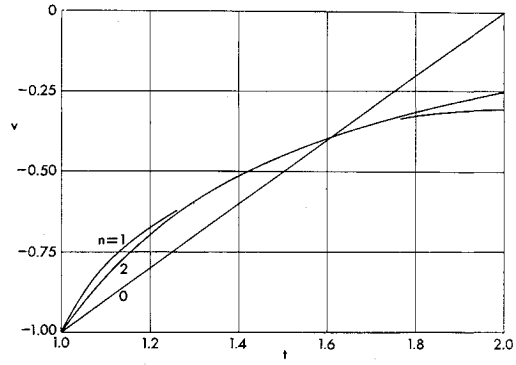

Fig. 5 The function $v(t)$.

If the constants k_1, k_2, k_3 are consistent with (11) and (12), then the linear combinations (10) are the desired solutions to (4). Once the perturbation functions are known, the approximate trajectory of the system is given by

$$x = x_* + \delta x \qquad y = y_* + \delta y \qquad z = z_* + \delta z$$
$$u = u_* + \delta u \qquad v = v_* + \delta v \tag{13}$$

and, in this way, the first iteration is completed. Next, the curve (13) is employed as the nominal curve for the second iteration, and the procedure is repeated until convergence is obtained.

3. Computational Results

Computations were performed with an IBM 7040 computer. The following nominal curve, satisfying the boundary conditions (2) and (3), was chosen for the first iteration:

$$x_* = 7t^2 - 6t \qquad y_* = 14t - 6 \qquad z_* = 14$$
$$u_* = t^2/2 - 2t + 5/2 \qquad v_* = t - 2 \tag{14}$$

Convergence was rapid. For instance, at station $t = 1.5$, no change occurred in the fifth significant figure after four iterations. The results are plotted in Figs. 1–5, in which n denotes the iteration number. Therefore, the curve $n = 0$ is the nominal curve (14), the curve $n = 1$ is that obtained after one iteration, and so on. For the functions $x(t)$, $y(t)$, $z(t)$, the curve $n = 4$ is not plotted, since it is extremely close to the curve $n = 3$. Analogously, for the functions $u(t)$, $v(t)$, the curve $n = 3$ is not plotted, since it is extremely close to the curve $n = 2$.

4. Conclusions

In the previous sections, the use of the method of particular solutions in quasilinearization was illustrated through a specific example, that of a nonlinear, fifth-order system subject to three initial conditions and two final conditions. The computational results show that the combination of quasilinearization with the method of particular solutions can be a powerful tool in solving nonlinear, two-point boundary-value problems. Provided the initial guess used in the iteration procedure is chosen with discretion, convergence to a solution is quite rapid, and the accuracy of the solution is limited only by the integration step size and the integration technique employed. For further applications of this technique to either uncontrolled systems or controlled systems, the reader should consult Refs. 4 and 5.

References

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Survey Paper Awards to be Repeated Next Year

At its meeting on October 23, 1968, the AIAA Board of Directors acted on a Publications Committee recommendation to authorize a second trial year for the four awards for the best survey papers in each of the AIAA journals (*AIAA Journal*, *Journal of Spacecraft and Rockets*, *Journal of Aircraft*, and *Journal of Hydronautics*). The winners of the first year's awards will be announced shortly by the Editors-in-Chief of the journals, and the awards will be presented at the 7th Aerospace Sciences Meeting in New York in January 1969.

The period for submission of papers for the second series of Survey Paper Awards will be October 1, 1968, through September 30, 1969. Like the current series, each award will carry an honorarium of $750, which will be presented at the subsequent Aerospace Sciences Meeting in January. Judges will be the Editors-in-Chief and the Associate Editors of the appropriate journal.

A competent survey paper is one which *critically* reviews all significant accomplishments in a given field, citing all prior pertinent literature. It thereby provides an informed basis from which the reader can proceed to his own evaluation of current activities in the field.

Jerry Grey
Vice-President, Publications